

# Generation of squeezed light using photorefractive degenerate two-wave mixing

Yajun Lü   Meijuan Wu   Ling-An Wu\*   Zheng Tang and Shiqun Li

Dept. of Modern Applied Physics, Tsinghua Univ., Beijing 100084, China

\*Institute of Physics, Academia Sinica, Beijing 100080, China

## Abstract

We present a quantum nonlinear model of two-wave mixing in a lossless photorefractive medium. A set of equations describing the quantum nonlinear coupling for the field operators is obtained. It is found that, to the second power term, the commutation relationship is maintained. The expectation values for the photon number concur with those of the classical electromagnetic theory when the initial intensities of the two beams are strong. We also calculate the quantum fluctuations of the two beams initially in the coherent state. With an appropriate choice of phase, quadrature squeezing or number state squeezing can be produced.

PACS numbers:42.654.Hw, 42.50.Dv, 42.50.Lc

## 1 Introduction

The photorefractive effect in electro-optic crystals, a phenomenon in which the local index of refraction is changed by the spatial variation of light intensity [1], has been studied extensively for its potential in many applications. The fundamental process may be described as follows. When the crystal is illuminated with a spatially modulated intensity pattern, free carriers (for example, electrons) are nonuniformly generated due to the photoionization of impurities (generally, which may be doped). The impurities that can be ionized and provide free carriers are called donors. Once these donors are ionized they can serve as trap sites which capture electrons. The electrons can be transported by diffusion or drift and become trapped at these sites. The trapped electrons can then be re-excited except for those in the dark region. Thus a space-charge separation is created, which leads to a space-charge field. Such a field induces a change in index of refraction via the Pockels effect (linear electro-optic effect), creating an index grating. The presence of such an index grating will in turn affect the propagation of these beams. Crystals such as  $LiNbO_3$ ,  $BaTiO_3$ ,  $SBN$ ,  $BSO$ ,  $GaAs$  and  $InP$ , are efficient media for the generation of the photorefractive effect with relatively low intensity level (eg.,  $1W/cm^2$ ).

Many different nonlinear optical phenomena in photorefractive media have been studied. These include wave mixing, phase conjugation, self-oscillation, photorefractive resonance, etc. The fundamental photorefractive process is two-wave mixing (TWM), in which two beams intersect inside a photorefractive media. A stationary index grating is formed which is spatially

shifted  $\pi/2$  relative to the intensity pattern. Such a spatial phase shift leads to nonreciprocal energy transfer when these two beams propagate through the medium. The basic classical electromagnetic theory explaining the nonlinear interaction involved is already well established. Much attention has been focused on its applications including photorefractive resonators, non-reciprocal transmission windows, self-pumped phase conjugators, laser beam clean-up, optical interconnection, etc. Although a number of cases of TWM have been analysed, a quantum theory is not available and photorefractive non-classical effects have not been discussed. In this paper we present, to our knowledge for the first time, a quantum treatment of two-wave coupling in a lossless photorefractive medium.

## 2 Quantum model of photorefractive TWM

A typical geometry for studying two-wave mixing is shown in Fig.1. Under certain circumstances, two beams of light can interact in a photorefractive crystal in such a manner that energy is transferred from one beam to the other. This process is also known as two-beam coupling. The signal and pump waves, of amplitudes  $A_s$  and  $A_p$ , respectively, interfere to form a nonuniform intensity distribution within the crystal. Due to the nonlinear response of the crystal, this nonuniform intensity distribution produces a refractive index grating within the material. However, this grating is displaced from the intensity distribution in the direction of the positive (or effective electrooptic coefficient) crystalline  $c$  axis. As a result of this phase shift, the light scattered from  $A_p$  into  $A_s$  interferes constructively with  $A_s$ , whereas the light scattered from  $A_s$  into  $A_p$  interferes destructively with  $A_p$ , and consequently the signal wave is amplified whereas the pump wave is attenuated.

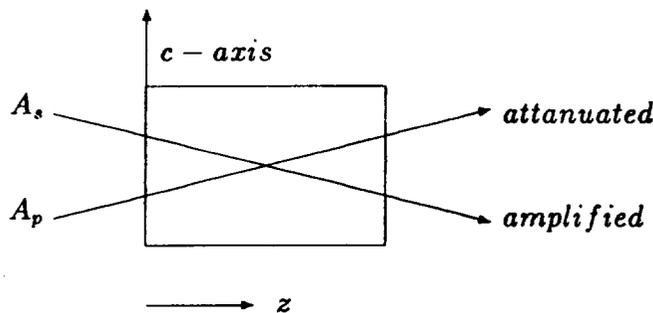


Fig.1. Typical geometry for studying two-beam coupling in a photorefractive crystal.

An ideal quantum model for degenerate two-wave mixing may be constructed as follows. Consider the effective interaction Hamiltonian

$$H_{eff} = \hbar\chi' A^\dagger (A^\dagger B + B^\dagger A) B + h.c. \quad (1)$$

where  $\chi'$  is the effective interaction coefficient for the nonlinear process, A and B are the Boson operators for two modes with frequency  $\omega_a = \omega_b$ . Factor  $(A^\dagger B + B^\dagger A)$  represents the interference of two modes [2]. The TWM can be understood from the following physical picture. Mode A is generated accompanied by the annihilation of mode B, due to scattering from the grating induced by the interference. In other words, mode B is "scattered" by the grating in the direction of beam A, to yield mode A, which is responsible for the energy coupling. The Heisengerg equations

of motion for the field operators A and B may be easily obtained from  $H_{eff}$ . Making the conversion  $z = vt$  for propagation along the  $z$ -axis at a velocity  $v$ , we can write the equations as

$$\frac{dA}{dz} = -2i\chi A^\dagger B^2 - i(\chi + \chi^*)AB^\dagger B \quad (2)$$

$$\frac{dB}{dz} = -2i\chi^* B^\dagger A^2 - i(\chi + \chi^*)BA^\dagger A \quad (3)$$

where  $\chi = \chi'/v$ . We find the field operators satisfy the Boson commutation rules. From the equations of motion for the photon number operators  $N_a$  and  $N_b$

$$\frac{dN_a}{dz} = -2i\chi A^{\dagger 2} B^2 + 2i\chi^* A^2 B^{\dagger 2} \quad (4)$$

$$\frac{dN_b}{dz} = 2i\chi A^{\dagger 2} B^2 - 2i\chi^* A^2 B^{\dagger 2} \quad (5)$$

we can show that the total photon number is constant throughout the process. In the short path approximation, the solutions of Eqs.(2) and (3) for the field operators with expansion up to the quadratic  $(\chi z)^2$  term is

$$\begin{aligned} A(z) = & a - 2i(\chi z)a^\dagger b^2 - iz(\chi + \chi^*)ab^\dagger b \\ & - |\chi z|^2 [(4a^\dagger a^2 + a)b^\dagger b - (2a^\dagger - a)b^{\dagger 2}b^2 + (a^{\dagger 2}b^2 + b^{\dagger 2}a^2)a + 2a^\dagger a^2] \\ & - z^2[\chi^2(a^{\dagger 2}ab^2 + b^{\dagger 2}b^2a/2 + b^\dagger ba/2) + \chi^*(a^3b^{\dagger 2} + ab^{\dagger 2}b^2/2 + ab^\dagger b/2)] \end{aligned} \quad (6)$$

$$\begin{aligned} B(z) = & b - 2i(\chi^* z)b^\dagger a^2 - iz(\chi + \chi^*)ba^\dagger a \\ & - |\chi z|^2 [(4b^\dagger b^2 + b)a^\dagger a - (2b^\dagger - b)a^{\dagger 2}a^2 + (b^{\dagger 2}a^2 + a^{\dagger 2}b^2)b + 2b^\dagger b^2] \\ & - z^2[\chi^2(b^{\dagger 2}ba^2 + a^{\dagger 2}a^2b/2 + a^\dagger ab/2) + \chi^*(b^3a^{\dagger 2} + ba^{\dagger 2}a^2/2 + ba^\dagger a/2)] \end{aligned} \quad (7)$$

where  $a$  and  $b$  are the input field operators, respectively. It may be seen that, to the quadratic term, the field operators still satisfy the commutation relation

$$[A(z), A^\dagger(z)] = 1, \quad [B(z), B^\dagger(z)] = 1 \quad (8)$$

In order to test our quantum model for photorefractive TWM, we may derive the expectation values for the photon number in each beam and verify if the result of the quantum calculation concur with those of the classical electromagnetic theory. When the two beams are initially in the coherent state  $|\alpha\rangle$  and  $|\beta\rangle$  with

$$\alpha = |\alpha| \exp(i\delta_a/2) \quad (9)$$

$$\beta = |\beta| \exp(i\delta_b/2) \quad (10)$$

$$\chi = |\chi| \exp(i\phi) \quad (11)$$

we obtain

$$\begin{aligned} \langle N_a \rangle = & |\alpha|^2 + 4\sin(\phi + \delta_b - \delta_a) |\chi z| |\alpha\beta|^2 + 4|\chi z|^2 (|\beta|^4 - |\alpha|^4) \\ & + 8|\chi z|^2 |\alpha\beta|^2 (|\beta|^2 - |\alpha|^2)[1 + \cos\phi \cos(\phi + \delta_b - \delta_a)] \end{aligned} \quad (12)$$

$$\langle N_b \rangle = |\alpha|^2 + |\beta|^2 - \langle N_a \rangle \quad (13)$$

where  $\phi$ ,  $\delta_b$  and  $\delta_a$  are phase angles, depending on the initial condition. We take  $\delta_b - \delta_a = \pi$  and write  $I_a = |\alpha|^2$ ,  $I_b = |\beta|^2$ , then obtain

$$\langle N_a \rangle = I_a - 4 \sin \phi |\chi z| I_a I_b + 4 |\chi z|^2 (I_b^2 - I_a^2) \quad (14)$$

$$\langle N_b \rangle = I_a + I_b - \langle N_a \rangle \quad (15)$$

According to the classical electromagnetic theory, the coupled equations for photorefractive TWDM can be written as [3]

$$\frac{dI_1}{dz} = -\gamma \frac{I_1 I_2}{I_1 + I_2} \quad (16)$$

$$\frac{dI_2}{dz} = \gamma \frac{I_1 I_2}{I_1 + I_2} \quad (17)$$

where  $I_1$  and  $I_2$  are the intensities of beams 1 and 2, respectively, and  $\gamma$  is the coupling constant with

$$\gamma = \frac{2\pi n_1}{\lambda \cos \theta} \sin \phi \quad (18)$$

Here  $n_1$  is the depth of index modulation related to the electro-optic coefficient,  $2\theta$  is the angle between the two beams inside the medium and  $\phi$  is the phase that indicates the degree to which the index grating is shifted spatially with respect to the light interference pattern.

By examining the coupled equations, we note that  $I_2$  is an increasing function of  $z$ , provided  $\gamma$  is positive. This indicates that the energy is flowing from beam 1 to beam 2. The direction of energy flow is determined by the sign of  $\gamma$ , which depends on the orientation of the crystal axis. The solution for the intensities  $I_1(z)$  and  $I_2(z)$  are

$$I_1(z) = I_1(0) \frac{I_1(0) + I_2(0) \exp(\gamma z)}{I_0} \quad (19)$$

$$I_2(z) = I_2(0) \exp(\gamma z) \frac{I_1(0) + I_2(0) \exp(\gamma z)}{I_0} \quad (20)$$

where  $I_1(0)$  and  $I_2(0)$  are the input intensities of beam 1 and beam 2, respectively, and  $I_0$  is the sum intensity with  $I_0 = I_1(0) + I_2(0)$ . In the short path approximation, the solutions can be expanded to the quadratic  $(\gamma z)^2$  term as

$$I_1(z) = I_1(0) - \gamma z \frac{I_1(0) I_2(0)}{I_0} + (\gamma z)^2 \frac{I_1(0) I_2(0)}{2I_0^2} [I_2(0) - I_1(0)] \quad (21)$$

$$I_2(z) = I_0 - I_1(z) \quad (22)$$

Comparing Eqs.(14) and (15) with Eqs.(21) and (22), we find that the results of the quantum theory are consistent with those of the classical theory, so long as  $I_1(0) = I_a$ ,  $I_2(0) = I_b$ ,  $\gamma = 4 |\chi| \sin \phi I_0$  and  $I_a \gg 1$ ,  $I_b \gg 1$ . When the input intensities of the two beams are strong, the effective Hamiltonian  $H_{eff}$  can give an accurate description of the energy exchange phenomenon in photorefractive two-wave mixing, as shown in Fig.2. We can

thus conclude that our quantum model for photorefractive TWM is reasonable and successful.

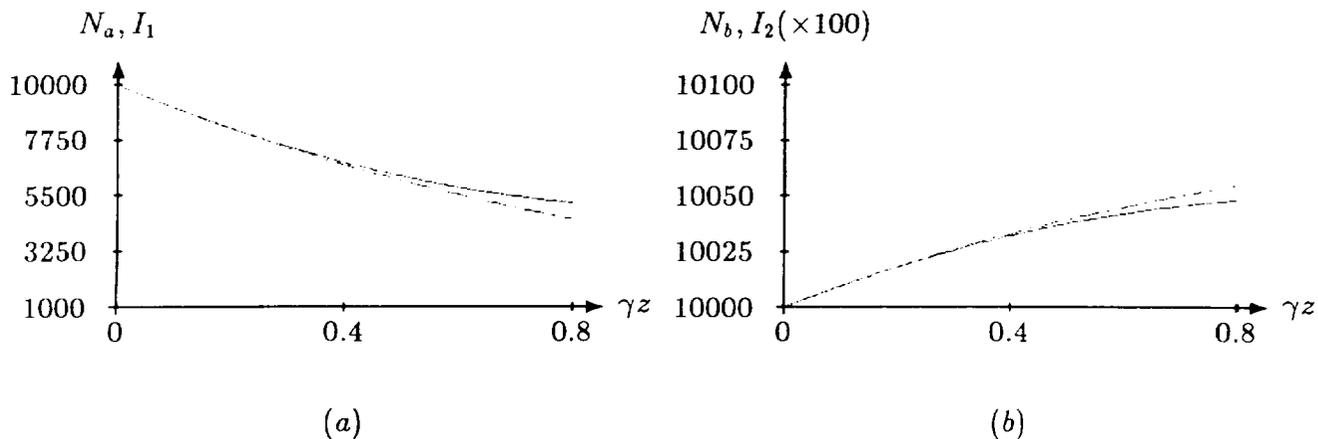


Fig.2. The intensities of two beams versus the effective interaction length ( $\gamma z$ ). Dashed curve: the intensities of the classical electromagnetic theory  $I_1$  and  $I_2$ , from Eqs.(19) and (20). Solid curve: the quantum average photon number  $N_a$  and  $N_b$ . The initial intensities  $I_1 = 10^4$ ,  $I_2 = 10^6$ , respectively.

### 3 Quantum statistic of photorefractive TWM

To discuss the photon number fluctuations of the quantized field we consider the variance  $\langle \Delta N_j^2(z) \rangle$  or the Fano factor

$$F_j(z) = \frac{\langle \Delta N_j^2(z) \rangle}{\langle \Delta N_j(z) \rangle} \quad (23)$$

where  $\langle \Delta N_j^2(z) \rangle = \langle N_j^2(z) \rangle - \langle N_j(z) \rangle^2$  and  $j = a, b$ . To obtain the above expression we need to find the expectation values for  $\langle \Delta N_a^2(z) \rangle$  and  $\langle \Delta N_b^2(z) \rangle$ . When the input fields are in the coherent state  $|\alpha\rangle$  and  $|\beta\rangle$ , after some tedious calculation we may obtain the expectation values

$$\langle \Delta N_a^2(z) \rangle = I_a - 8 \sin \phi |\chi z| I_a I_b + 8 |\chi z|^2 I_a [2I_a^2 I_b \cos(2\phi) + 2I_a I_b^2 (1 + \sin^2 \phi) - I_b^2 - I_a^2] \quad (24)$$

$$\langle \Delta N_b^2(z) \rangle = I_b + 8 \sin \phi |\chi z| I_a I_b + 8 |\chi z|^2 I_b [2I_b^2 I_a \cos(2\phi) + 2I_b I_a^2 (1 + \sin^2 \phi) - I_a^2 - I_b^2] \quad (25)$$

where we have taken  $\delta_b - \delta_a = \pi$ . Here  $\phi = \pm \frac{\pi}{2}$  corresponds to the maximum energy coupling between the two beams. Eqs.(14) and (15) show that the energy flows from beam A to beam B when  $\phi \in [0, \pi]$ . This indicates that A is the pump beam and B the signal beam. Let  $\phi = \frac{\pi}{2}$ , we rewrite Eqs.(24) and (25) as

$$\langle \Delta N_a^2(z) \rangle = I_a - 8 |\chi z| I_a I_b + 8 |\chi z|^2 I_a [-2I_a^2 I_b + 4I_a I_b^2 + I_b^2 - I_a^2] \quad (26)$$

$$\langle \Delta N_b^2(z) \rangle = I_b + 8 |\chi z| I_a I_b + 8 |\chi z|^2 I_b [-2I_b^2 I_a + 4I_b I_a^2 + I_a^2 - I_b^2] \quad (27)$$

The Fano factor plotted against the effective interaction length  $\gamma z$  is shown in Fig.3, where  $\gamma = 4 |\chi| \sin \phi I_0$ . We see that the pump mode A can be in a squeezed number state, whereas the signal mode B becomes super-Poissonian at the same time.

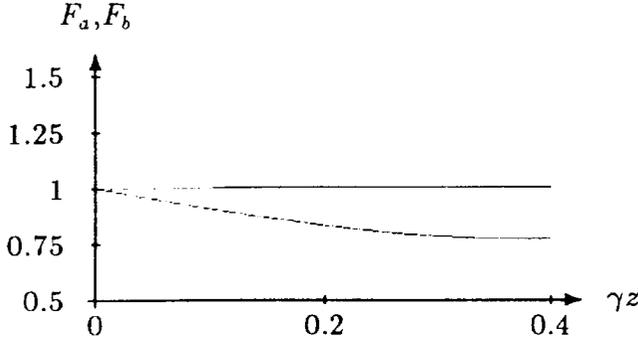


Fig.3. Fano factors of beams A and B versus effective interaction length  $\gamma z$  for coherent state inputs. Dashed curve:  $F_a$ , solid curve:  $F_b$ . The initial intensity ratio  $m = \frac{I_a(0)}{I_b(0)} = \frac{1}{100}$ .  $F_a$  shows the sub-Poissonian statistical feature.

Moreover, the signal mode can never becomes squeezed(at least for our solution expanded to the second order). The degree of squeezing in the pump mode depends on the initial intensity ratio  $m$  (here we define  $m = \frac{I_a(0)}{I_b(0)}$ ). If  $m$  is large (for example, 100), then the degree of squeezing will be very small (in the short path approximation). This is reasonable as the energy coupling has little effect on the intensity of the pump modes, so the quantum fluctuations will not be reduced greatly.

The quadrature phase amplitudes of the two beams are defined as

$$X_a = \frac{A(z)+A^\dagger(z)}{2} \quad Y_a = \frac{A(z)-A^\dagger(z)}{2i} \quad (28)$$

$$X_b = \frac{B(z)+B^\dagger(z)}{2} \quad Y_b = \frac{B(z)-B^\dagger(z)}{2i} \quad (29)$$

When the input field are in the coherent state, the field variances may be determined explicitly to be

$$\langle \Delta X_a^2(z) \rangle = \frac{1}{4} + |\chi z| I_b \sin(\phi + \delta_b) + 2 |\chi z|^2 I_b [I_b + I_a \cos^2 \phi - I_a \cos \delta_a (1 + \cos^2 \phi) - (2I_a + I_b) \cos \phi \cos(\phi + \delta_b)] \quad (30)$$

$$\langle \Delta Y_a^2(z) \rangle = \frac{1}{4} - |\chi z| I_b \sin(\phi + \delta_b) + 2 |\chi z|^2 I_b [I_b + I_a \cos^2 \phi + I_a \cos \delta_a (1 + \cos^2 \phi) + (2I_a + I_b) \cos \phi \cos(\phi + \delta_b)] \quad (31)$$

$$\langle \Delta X_b^2(z) \rangle = \frac{1}{4} + |\chi z| I_a \sin(\delta_a - \phi) + 2 |\chi z|^2 I_a [I_a + I_b \cos^2 \phi - I_b \cos \delta_b (1 + \cos^2 \phi) - (2I_b + I_a) \cos \phi \cos(\delta_a - \phi)] \quad (32)$$

$$\langle \Delta Y_b^2(z) \rangle = \frac{1}{4} - |\chi z| I_a \sin(\delta_a - \phi) + 2 |\chi z|^2 I_a [I_a + I_b \cos^2 \phi + I_b \cos \delta_b (1 + \cos^2 \phi) + (2I_b + I_a) \cos \phi \cos(\delta_a - \phi)] \quad (33)$$

With an appropriate choice of phase, both modes can produce quadrature squeezing. For example, when  $\phi = \pi/2$ ,  $\delta_b = \pi$  and  $\delta_a = 0$ , it is obvious that  $\langle \Delta X_a^2(z) \rangle$  and  $\langle \Delta X_b^2(z) \rangle$  may be less than  $\frac{1}{4}$  in the short path approximation. The variances plotted against the effective interaction  $\gamma z$  are shown in Fig.4. We see that both modes can be in the squeezed state. Furthermore, there is strong dependence on which of the input modes is strong. The degree of squeezing in the pump mode is great when the initial intensity ratio  $m$  is small, as shown in Fig.4. In the reverse case,

if the pump mode A is strong, then the degree of squeezing in the signal mode B will be great.

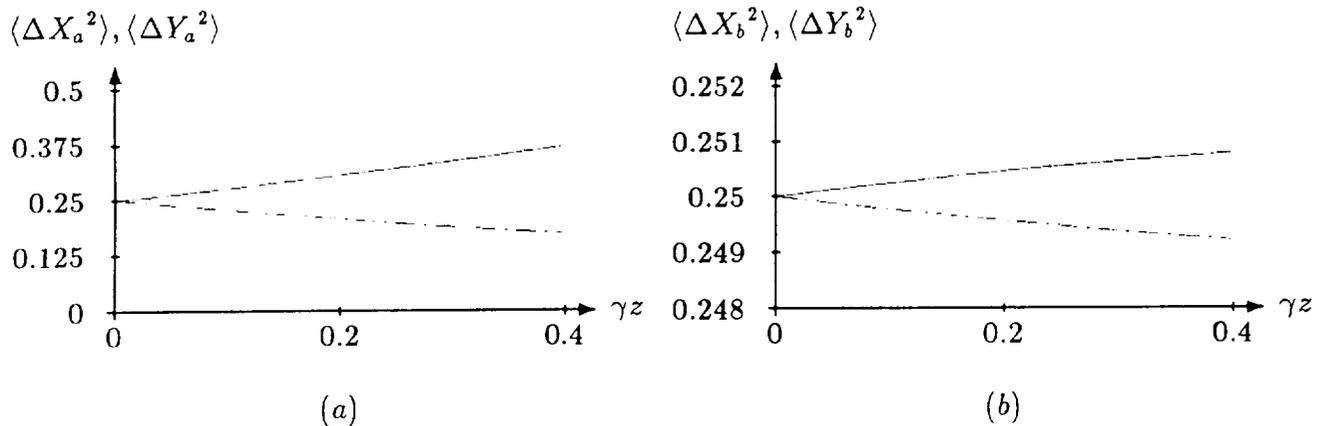


Fig.4. The variances  $\langle \Delta X_a^2 \rangle$ ,  $\langle \Delta Y_a^2 \rangle$ ,  $\langle \Delta X_b^2 \rangle$ , and  $\langle \Delta Y_b^2 \rangle$  when both input field are initially in the coherent state, with  $m = \frac{1}{100}$ . Dashed curves: the variances of X component, which show quadrature squeezing. Solid curves: the variances of Y component.

## 4 Conclusion

In conclusion, we have presented a quantum model of photorefractive TWM, which can well describe the energy exchange phenomenon in TWM. A set of coupled mode equations is obtained and solved in the short path approximation. We have also calculated the quantum fluctuations of the two modes and find that when both modes are initially in the coherent state, the pump beam can become sup-Poissonian, due to the photon flux in the energy transfer. The same qualitative result was also obtained in our previous approach from a set of simplified field equations [4]. With an appropriate choice of phase relationship, quadrature squeezing can be produced.

## References

- [1] P. Yeh, *Introduction to Photorefractive Nonlinear Optics* (Wiley, New York, 1993).
- [2] J. Perina, *Quantum Statistic of Linear and Nonlinear Optical Phenomena* (Kluwer Academic publishers, 1991).
- [3] P. Yeh, *IEEE J. Quantum Electron*, **3**, 484 (1989).
- [4] Tang Zheng, Lü Yajun, Wu Meijuan, Li Shiqun and Wu Ling-An, 6th National Conf. on Quantum Optics, Jiangxi, China, Oct, 1994 (in chinese).

